Centrum Fizyki Teoretycznej PAN

Vortices in the Many-Body Excited States of Interacting Bosons in Two Dimensions

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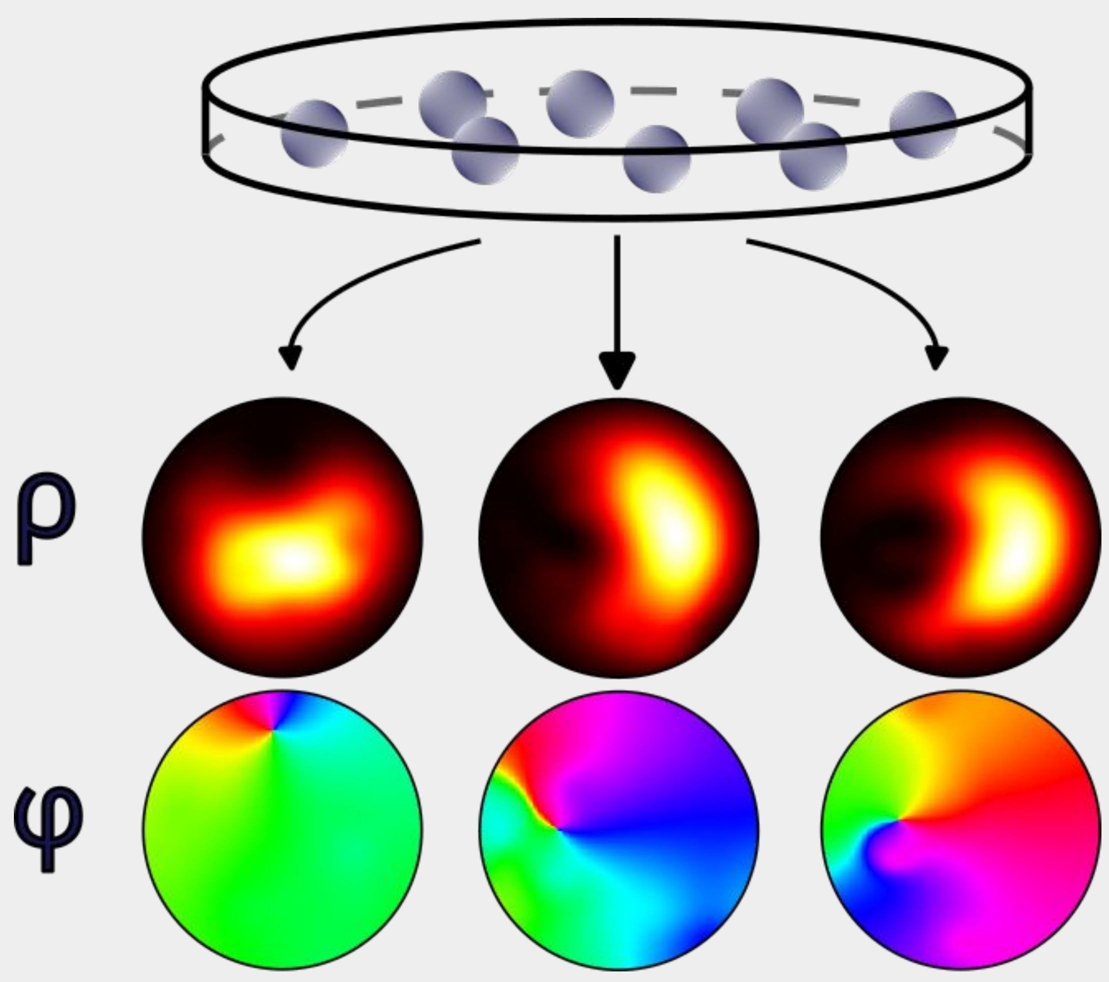
Overview

Quantum vortices are key features in two-dimensional **quantum many-body systems**, yet they are often described using single-particle approximations such as **mean-field theory**. In this work, we explore how **vortices** emerge directly from the eigenstates of an **N-body Hamiltonian** describing interacting particles confined in a disk. Inspired by soliton studies in 1D systems [2], we examine states revealed through sequential position measurements of individual particles. Remarkably, these many-body states exhibit densities, phases, and energies that closely mirror those of mean-field vortices.

$$\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_N)$$



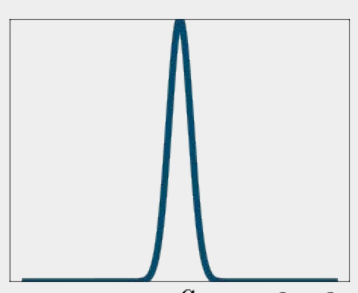
Numerics

Many-Body

Potential: gaussian - numerically convergent

Method: exact diagonalization with importance truncation [3]

$$\hat{H} = -\frac{\hbar^2}{2m}\sum_i \nabla_i^2 + \frac{1}{2}\sum_{i\neq j} V(|\boldsymbol{r}_i - \boldsymbol{r}_j|)$$



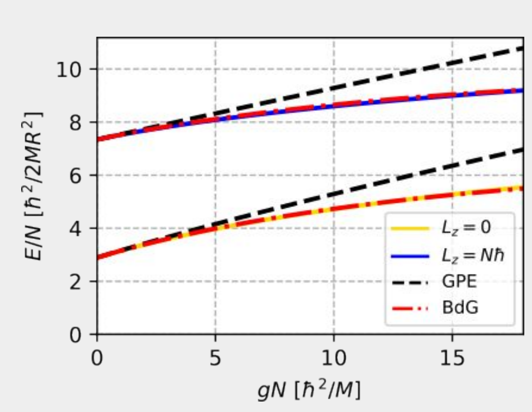
$$V(\boldsymbol{r}) = \frac{g}{\pi\sigma^2} e^{-r^2/\sigma^2}$$

Mean-field

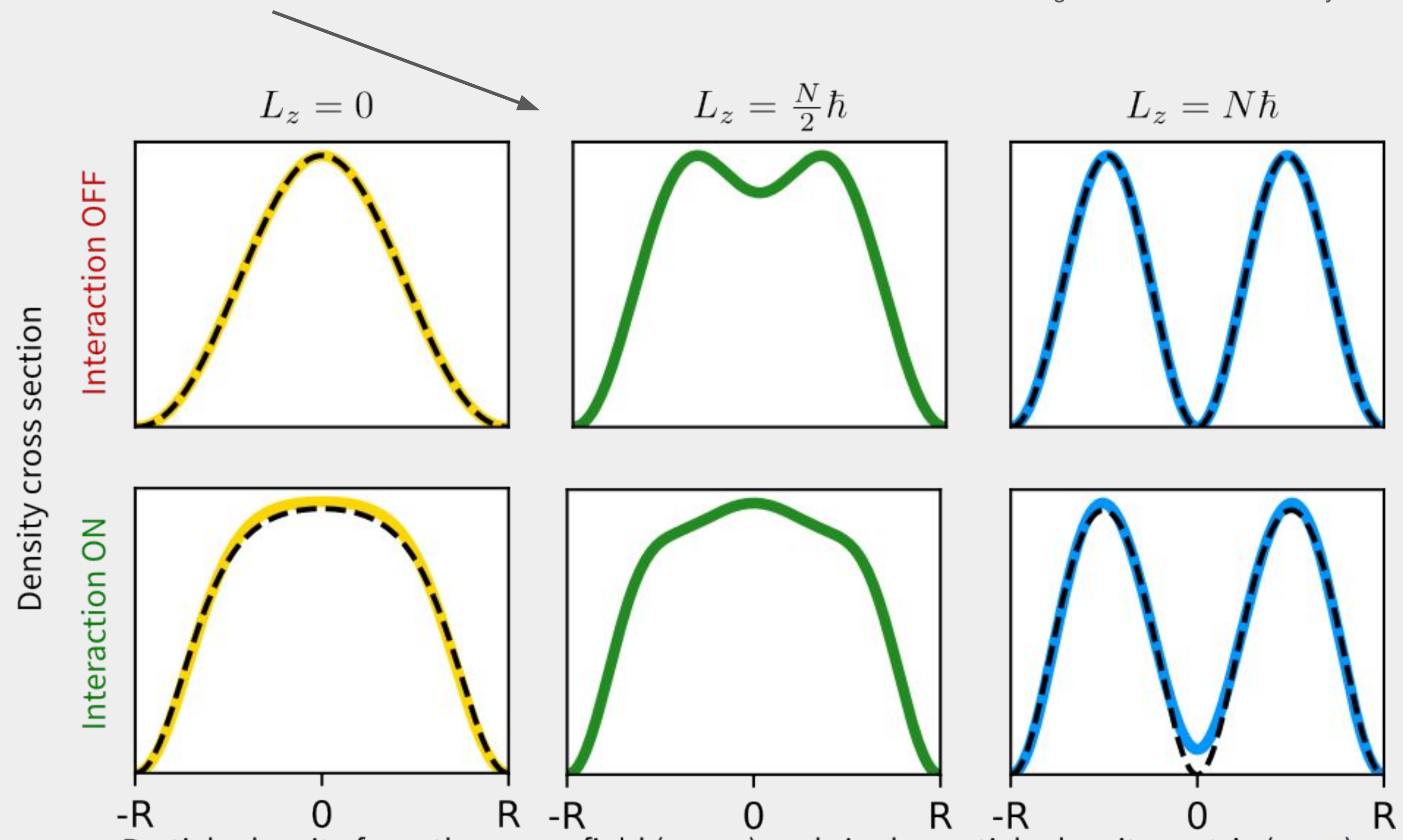
The Gross-Pitaevskii equation

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + (N-1)\left(V * |\phi|^2\right)\right)\phi = \mu\phi$$

We compare: densities, energy



Energy from many-body, mean-field and Bogoliubov-de Gennes analysis



Particle density from the mean-field (−−−) and single-particle density matrix (▬)

Measurement

We sequentially measure the positions of N−1 particles using a recursive procedure. These measured positions serve as parameters in the conditional wave function of the remaining particle, effectively revealing the presence and structure of vortices.

$$\rho_1(\boldsymbol{r}) = \int |\Psi(\boldsymbol{r}, \boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_{N-1})|^2\, \mathrm{d}\boldsymbol{r}_1 \ldots \mathrm{d}\boldsymbol{r}_{N-1} \longrightarrow \bar{\boldsymbol{r}}_1$$

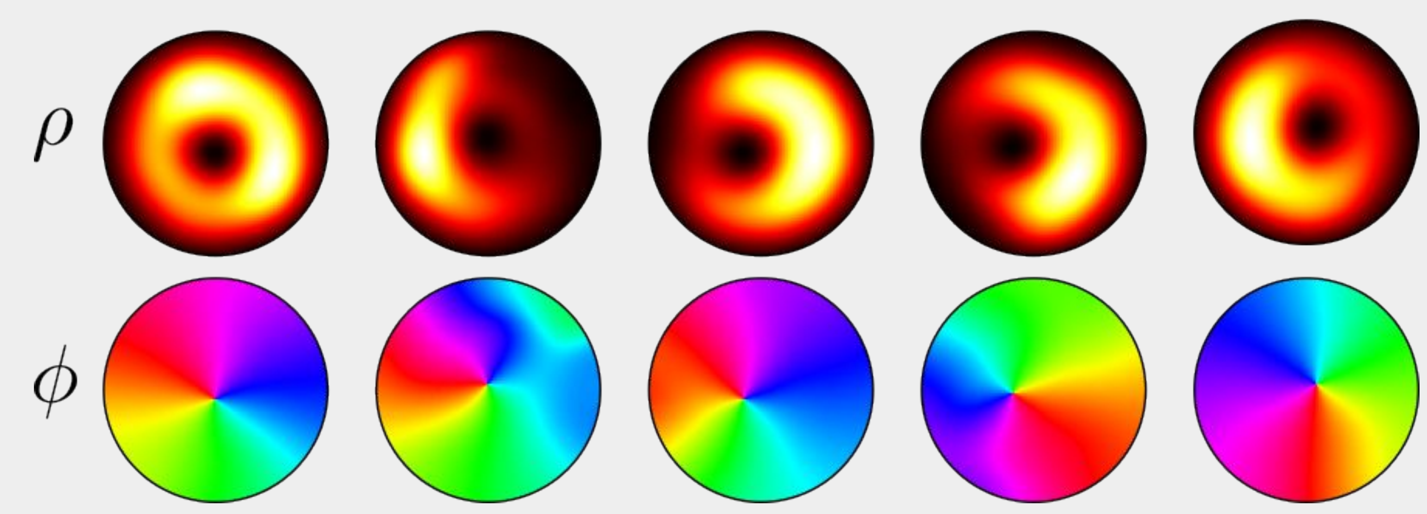
$$\rho_2(\boldsymbol{r}) = \int |\Psi(\boldsymbol{r}, \bar{\boldsymbol{r}}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_{N-1})|^2\, \mathrm{d}\boldsymbol{r}_2 \ldots \mathrm{d}\boldsymbol{r}_{N-1} \longrightarrow \bar{\boldsymbol{r}}_2$$

$$\vdots$$

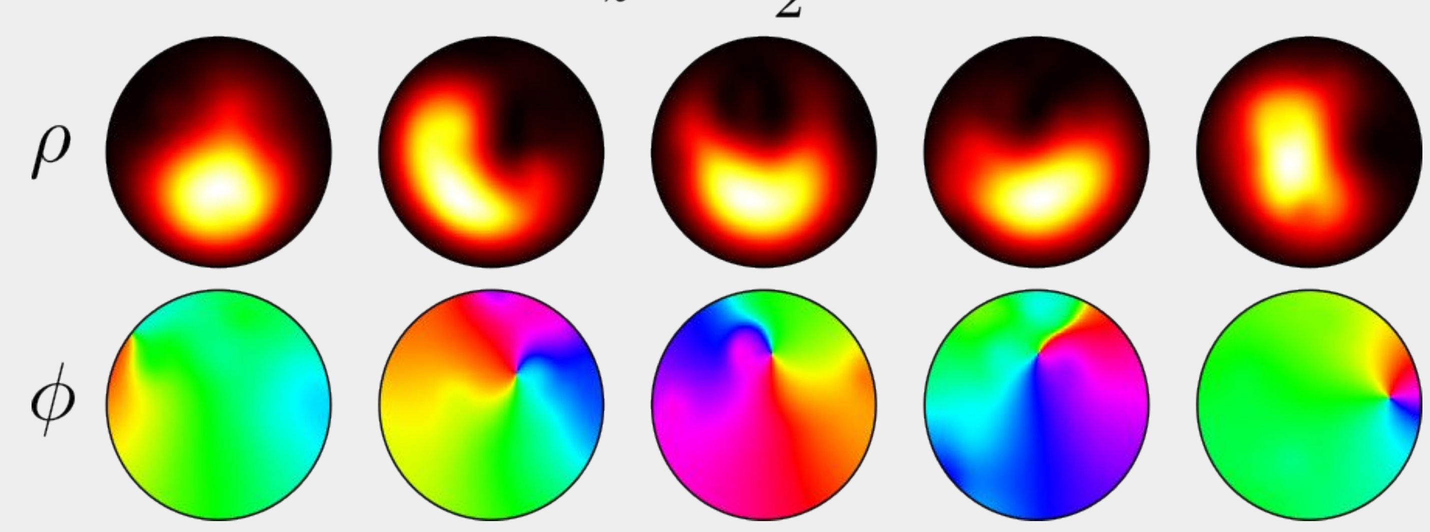
$$\bar{\boldsymbol{r}}_1, \bar{\boldsymbol{r}}_2, \ldots, \bar{\boldsymbol{r}}_{N-1}$$

$$\psi_{\text{cond}}(\boldsymbol{r}) = \Psi(\boldsymbol{r}, \bar{\boldsymbol{r}}_1, \ldots, \bar{\boldsymbol{r}}_{N-1})$$

$$L_z = N\hbar$$



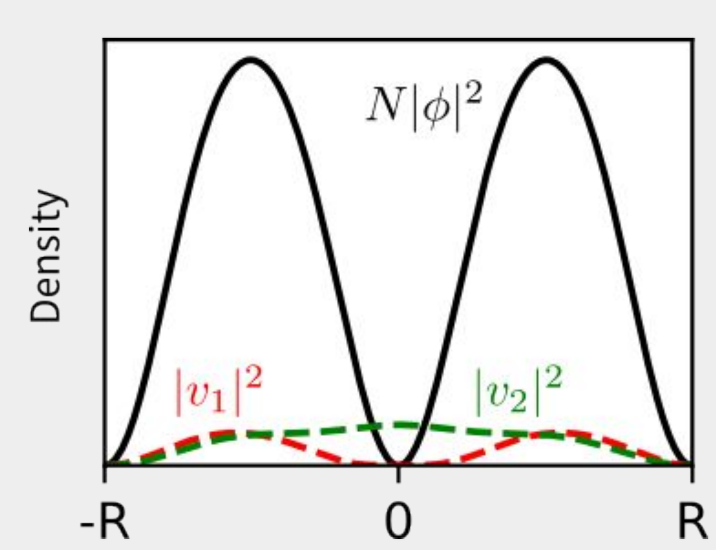
$$L_z = \frac{N}{2}\hbar$$



We found that during position measurements in the 2D system in the yrast state, mean-field vortices emerge.

Bogoliubov-de Gennes analysis

To further support our findings, we perform a number-conserving Bogoliubov–de Gennes analysis. The macroscopically occupied single orbital is the mean-field solution. This approach allows us to identify the modes into which the condensate is depleted and to compute energy corrections beyond the mean-field approximation.



Mean-field density and two mostly occupied Bogoliubov-de Gennes modes

$$\begin{pmatrix} \hat{H}_{\mathrm{GP}} + \phi\hat{U}^* & \phi\hat{U} \\ -\phi^*\hat{U}^* & -\hat{H}_{\mathrm{GP}} - \phi^*\hat{U} \end{pmatrix}\begin{pmatrix} u_n \\ v_n \end{pmatrix} = \epsilon_n \begin{pmatrix} u_n \\ v_n \end{pmatrix}$$

[1] M. Ślusarczyk and K. Pawłowski, Vortices in the many-body excited states of interacting bosons in two dimensions, Phys. Rev. A 111, 053314 (2025).

[2] A. Syrwid and K. Sacha, Lieb-liniger model: Emergence of dark solitons in the course of measurements of particle positions, Physical Review A 92, 032110 (2015).

[3] R. Roth, Importance truncation for large-scale configuration interaction approaches, Phys. Rev. C 79, 064324 (2009).

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